Instructions: Complete each of the following as practice.

1. Show each of the following sets is a vector space.
   (a) $U = \{(x, y, z) \in \mathbb{R}^3 : x + 2y - 3z = 0\}$ with the usual vector addition and scalar multiplication.
   (b) $V = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$ with the usual matrix addition and scalar multiplication.
   (c) $W = \{a + bx : a, b \in \mathbb{R}\}$ with the usual polynomial addition and scalar multiplication.
2. Show each of the following sets is NOT a vector space. Which vector space axiom(s) does each fail?
   (a) $U = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$ with the usual vector addition and scalar multiplication.
   (b) $V = \left\{ \begin{bmatrix} \pi & b \\ c & 0 \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$ with the usual matrix addition and scalar multiplication.
   (c) $W = \{p(x) \in \mathcal{P}_2(\mathbb{R}) : p(0) = -3\}$ with the usual polynomial addition and scalar multiplication.
   (d) $X = \mathbb{R}^n$ with the usual vector addition and $c \cdot v = \vec{0}$ for all $v \in X$.
3. Let V be an arbitrary (real) vector space. Prove the following.
   (a) For all $v \in V$ we have $0 \cdot v = 0_V$.
   (b) For all $c \in \mathbb{R}$ we have $c \cdot 0_V = 0_V$.
   (c) For all $v \in V$ we have $-1 \cdot v = -v$ (i.e. the additive inverse of v is $-1 \cdot v$).
4. Is the set of rational numbers a (real) vector space under addition and scalar multiplication? Why or why not?
5. Let S be an arbitrary set and let V be a vector space. Prove that the set $\mathrm{Func}(S, V)$ is a vector space under the operations $(f + g)(x) = f(x) + g(x)$ and $(c \cdot f)(x) = cf(x)$.
6. For further exercises, see the following (note: this list may break with future versions of these textbooks).
   (a) Beezer NONE
   (b) Hefferon page 92 (problems 1.17 – 1.45)
   (c) Matthews NONE